
SEMESTER 1 ASSESSMENT PAPER 2022/23

TITLE: AEROSPACE CONTROL DESIGN

DURATION – 2 HOUR

This paper contains **FOUR** questions.

Answer **ALL** questions in **Section A** and **only TWO** questions in **Section B.**

Section A carries 40% of the total marks for the exam paper.

Section B carries 60% of the total marks for the exam paper.

An outline marking scheme is shown in brackets to the right of each question. Note that marks will only be awarded when appropriate working is given. All solutions should be hand-written and all steps should be shown to receive full credit. Provide explanations for every answer and indicate the unit(s) used in **ALL** calculations.

Additional generic rubric will apply in the event that the exam is moved online.

Only University approved calculators may be used.

A foreign language direct 'Word to Word' translation dictionary (paper version ONLY) is permitted, provided it contains no notes, additions or annotations.

SECTION A

A.

(i) Using the block manipulation techniques, reduce the block diagram shown in Figure A-1 to a single block and write the overall transfer function $C(s)/R(s)$. Show all the steps performed.

[6 marks]

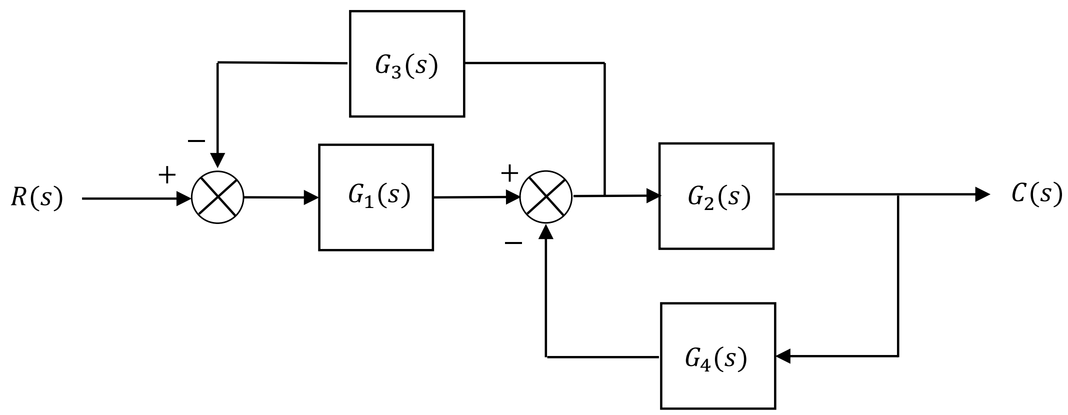

Figure A-1

(ii) Find a transfer function of a second order system:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

that yields a 72.9% overshoot and a 5%-settling time 3 seconds.

[4 marks]

(iii) Write the input-output relationship of a PID controller in the time domain and determine the corresponding transfer function. Briefly describe the role of each term in the PID controller.

[3 marks]

TURN OVER

(iv) Give a definition of the transfer function. The output of a system given by transfer function $G(s)$ to a unit step input $u(s) = 1/s$ is expressed in the time domain for $t \geq 0$ as:

$$y(t) = 3 - 5e^{-t} + 2e^{-2t}$$

Determine the transfer function $G(s)$. [5 marks]

(v) (a) Linearise the differential equation:

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + 4x^2 = \exp(-t)$$

around the steady state equilibrium point $t_0 = 0, x_0 = 1/2$.
Hint: $\exp(x) = 1 + x + x^2/2 + \cdots$.

(b) Find the transfer function of a system with dynamics equivalent to the linearised equation obtained in (a).

[7 marks]

(vi) Find the natural frequency (ω_n), damping factor (ζ), peak time (T_p), settling time (T_s), and percentage overshoot (PO) to a step input for the system given by the transfer function:

$$G(s) = \frac{50}{s^2 + 2s + 25}$$

[5 marks]

(vii) Find a state space form for the system described by the transfer function:

$$G(s) = \frac{s + 3}{s^2 + 4s + 2}$$

[6 marks]

(viii) Determine the gain and phase of the system at frequency $\omega = 1$ rad/sec:

$$G(s) = \frac{s^3 + 2s^2}{s^4 + s^3 + 1}$$

[4 marks]

SECTION B

B1. You have been asked to perform system identification to work out the transfer function of the system using the root locus diagram in Figure B1-1.

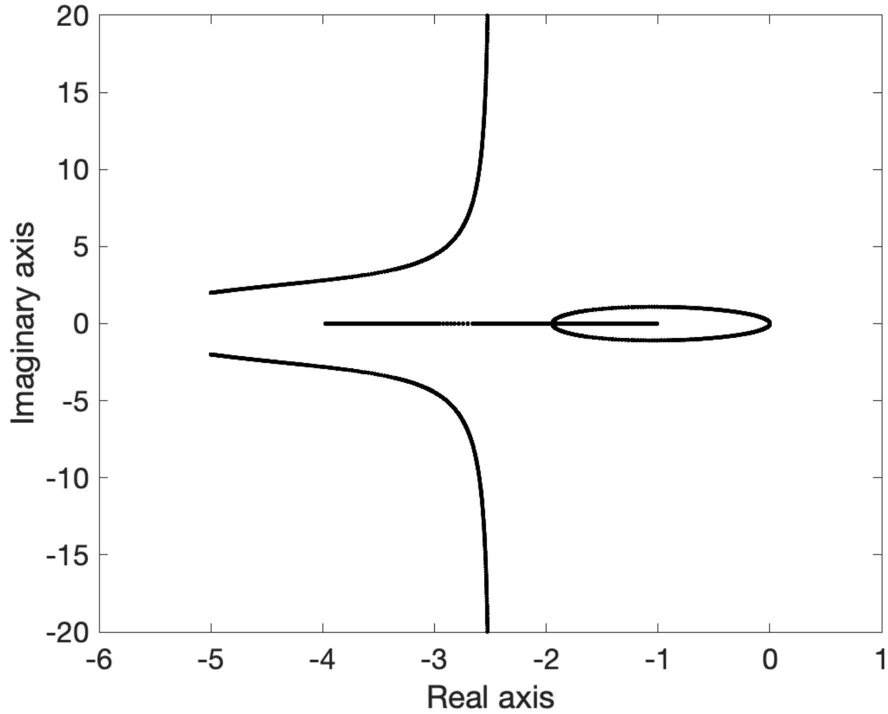

Figure B1-1

[30 marks total]

(i) Identify the transfer function from the root locus diagram in Figure B1-1. Hint: To solve the problem, break down the system using 3 steps. Step 1: consider the number of asymptotes, their angles and where they intersect the real axis. Step 2: identify which parts of the root loci are on the real axis. Step 3: Identify any points of breakaway or arrival from the real axis. In other words, work backwards from the answer to work out the original transfer function.

[10 marks]

(ii) What is the steady-state error to a ramp input?

[4 marks]

TURN OVER

(iii) Explain why adding a PI controller might not have much benefit for this system.

[4 marks]

(iv) Let's add a PD controller of the form $G_c = s + 2$. Draw the new root locus diagram.

[8 marks]

(v) Using your result to part (iv), what benefit does a PD controller have over a P controller?

[4 marks]

TURN OVER

B2. Consider an automatic flight control system of an F-8 aircraft. The flight dynamics are given by the following set of ordinary differential equations

$$\dot{x} = -x + z - x^2 z - y^2$$
$$\dot{y} = z$$
$$\dot{z} = -4(x+1)^2 - z + u$$

where x is the angle of attack (rad), y is the pitch angle (rad), z is the pitch rate (rad/s) and u is the control.

[30 marks total]

(i) Find all of the equilibrium points for this system setting the control term to zero, i.e. $u = 0$.

[4 marks]

(ii) Linearise the system around the equilibrium point $(x_0, y_0, z_0) = (-1,1,0)$.

[10 marks]

(iii) Express the linearised system, which includes the control term, in state-space form.

[3 marks]

(iv) Is the system controllable?

[5 marks]

(v) Design a proportional controller using pole placement so that the closed-loop poles are at $(-1,0,-3)$. Hint: Use a gain matrix of the following form $K = (0,0,k_3)$.

[6 marks]

(vi) If the control term also appeared in the equation for $\dot{x}$, does pole placement still work?

[2 marks]

B3. You have been tasked with designing a damping system that reduces vibrations during take-off. The transfer function is not known, but an engineer has been able to produce the following Bode plots in Figure B3-1.

[30 marks total]

(i) Identify the transfer function from the Bode plot. Please provide a sufficient number of steps for how you arrived at your answer.

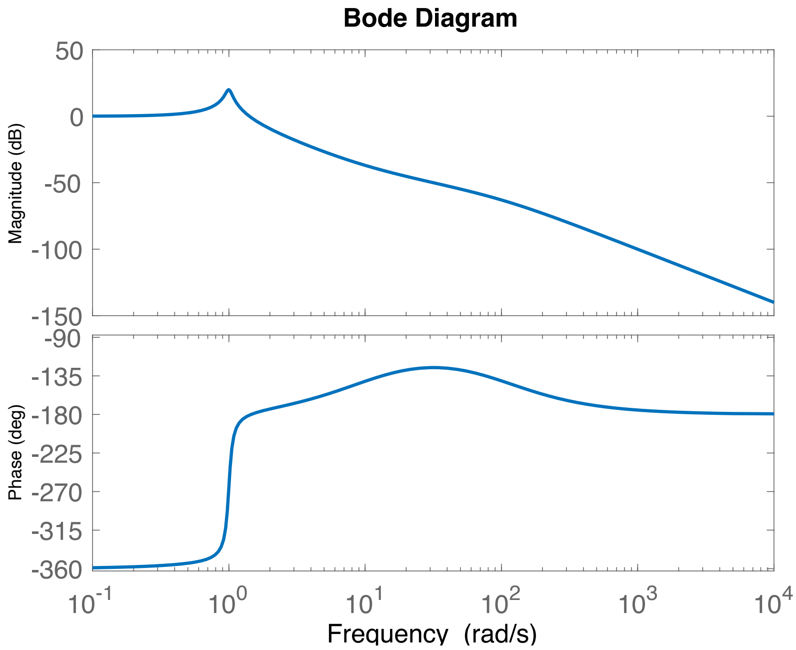

Figure B3-1

[12 marks]

(ii) Draw the Nyquist plot for this system and use it to argue if P control is sufficient to stabilise the system.

[10 marks]

(iii) Propose a controller that would reduce amplification at low frequencies.

[4 marks]

TURN OVER

(iv) **<u>Sketch</u>** the modified Bode diagram that implements the controller you designed in part (iii). There is no need to be quantitatively accurate.

[4 marks]

END OF PAPER