

1)

a)  $\frac{K_p(1+T_d s)}{s^3 + 5s^2 + 12s - 18}$

$\frac{s^3 + 5s^2 + 12s + 18}{s^3 + 5s^2 + 12s - 18}$

$\frac{K_p(1+T_d s)}{s^3 + 5s^2 + 12s - 18}$

$\frac{K_p(1+T_d s)}{s^3 + 5s^2 + 12s - 18 + K_p(1+T_d s)}$

Set  $T_d = 0.5$

$K_p + 0.5K_p s$

$s^3 + 5s^2 + (12 + 0.5K_p)s + K_p - 18$

b) ~~This equation~~ The characteristic polynomial has no roots at 0, so it is a type 0 system. There is a steady state error of  $\frac{1}{1+K_p}$  to a step input. Increasing  $K_p$  or adding integral control will solve this.

c) 
$$\begin{array}{ccc|c} s^3 & 1 & 12 + 0.5K_p & 0 \\ s^2 & 5 & K_p - 18 & 0 \\ s^1 & 1.5K_p + 78 & 0 & 0 \\ s^0 & 18 - K_p & 0 & 0 \end{array}$$

~~$$\begin{array}{c|c} 5 & K_p - 18 \\ \hline 1.5K_p & \end{array}$$~~

$$\begin{array}{c|c} 1 & 12 + 0.5K_p \\ \hline 5 & K_p - 18 \end{array}$$

$$\frac{-5(12 + 0.5K_p) + K_p - 18}{5} = 5$$

$18 - K_p > 0$  when  $K_p > 18$

$1.5K_p + 78 > 0$  when  $K_p > -52$

The system is stable when  $K_p > 18$

$$\begin{array}{c|c} 5 & K_p - 18 \\ \hline 1.5K_p + 78 & 0 \end{array}$$

$1.5K_p + 78$

$$\frac{(K_p - 18)(1.5K_p + 78)}{1.5K_p + 78}$$

2)

$$a) A^2 = \begin{bmatrix} 100 & -11 & 2k_i \\ 0 & 1 & -2k_i \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -10 & 1 & 0 \\ 100 & -11 & 2k_i \end{bmatrix}$$

rank = 3  $\therefore$  fully observable

$$b) G = \frac{V(s)}{U(s)} = C [I_s - A]^{-1} B$$

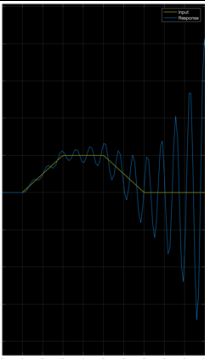
$$I_s - A = \begin{bmatrix} s+10 & -1 & 0 \\ 0 & s+1 & 2k \\ 0 & 0 & s \end{bmatrix}$$

$$[I_s - A]^{-1} = \begin{bmatrix} \frac{1}{s+10} & \frac{1}{(s+1)(s+10)} & + \frac{2k}{s(s+1)(s+10)} \\ 0 & \frac{1}{s+1} & - \frac{2k}{s(s+1)} \\ 0 & 0 & \frac{1}{s} \end{bmatrix}$$

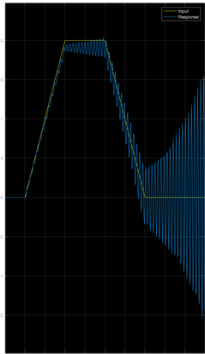
thank you MATLAB

$$C [I_s - A]^{-1} = \begin{bmatrix} \frac{1}{s+10} & \frac{1}{(s+1)(s+10)} & \frac{2k}{s(s+1)(s+10)} \end{bmatrix} = \text{temp}$$

$$\text{temp} B = \frac{2k_p}{(s+1)(s+10)} + \frac{2k_i}{s(s+1)(s+10)} = \frac{2s k_p + 2k_i}{s(s+1)(s+10)}$$



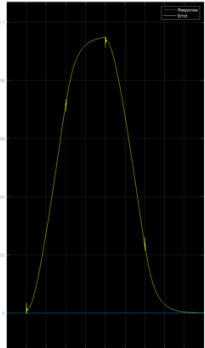
The response oscillates at increasing amplitude, so the drone will crash.



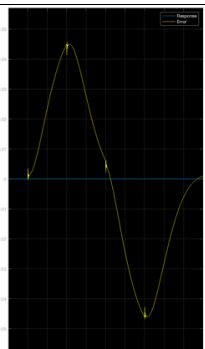
The drone will still crash



This stabilises the trajectory



This shows the error without integral control



This shows the error with integral control. The error decreases, visible on the scale

