

SESA6085 - Tutorial Workbook

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Overview

The following document provides a series of tutorial examples covering the mathematical modelling aspects of SESA6085. This is intended to act as an aid to preparation for any summative assessment and to further your understanding of the material.

Basic Probability Theory

Q1.1:

A test set has a 98% probability of correctly classifying a faulty item as defective and a 4% probability of classifying a good item as defective. If in a batch of items tested 3% are actually defective, what is the probability that when an item is classified as defective, it is truly defective?

Q1.2:

In the test firing of a missile, there are some events that are known to cause the missile to fail to reach its target. These events are listed in Table 1 together with their approximate probabilities of occurrence during a flight and the probability of failure if each event occurs. Calculate the probability of each of these events being the cause in the event of a missile failing to reach its target.

Table 1:

Event	Probability $P(A_i)$	$P(F A_i)$
Cloud reflection (A_1)	0.004	0.06
Precipitation (A_2)	0.011	0.03
Target evasion (A_3)	0.007	0.09
Electronic countermeasures (A_4)	0.05	0.07

Continuous Distributions

Note that Table 12 in the Appendix contains the standard normal distribution which can be used to help solve some of the below equations.

Q2.1:

Using Excel, Matlab, Python or similar, create a plot of the PDF, CDF and reliability function of normal distribution with a $\mu = 5$ and $\sigma = 1$ between $t = 0$ and $t = 10$.

Q2.2:

The life of an incandescent lamp is s normally distributed, with mean 1200hrs and standard deviation 200hrs. Calculate the probability that a lamp will last (a) at least 800hrs (b) at least 1600hrs.

Q2.3:

A designer could replace the incandescent bulb from the previous question with a different bulb. He has three options with lives described by the following normal distributions.

- $\mu_1 = 1300, \sigma_1 = 400$
- $\mu_2 = 1600, \sigma_2 = 400$
- $\mu_3 = 1300, \sigma_3 = 200$

If the aim is to improve reliability at 800hrs which bulb should the designer select?

Q2.4:

After a turbine blade has been manufactured the angle of its firtree face is measured. Previous data suggests that the angle of the face varies according to a three parameter Weibull distribution with $\beta = 3.5$, $\eta = 6.0$ and $\gamma = 15^\circ$. A turbine blade is rejected if the angle of it's firtree is less than 18° and greater than 23° . What is the probability that a turbine blade is rejected?

Q2.5:

The reliability of a communications satellite is described by a Weibull distribution with parameters, $\beta = 1.75$, $\eta = 4.0$ and $\gamma = 9$, with units of years. The satellite operator intends to replace the system when it's reliability drops to 30%. How many years after the system is first launched should it be replaced?

Q2.6:

A number of servos have been tested and MLE used to fit a log-normal distribution to the failure data. The resulting distribution has $\mu = 5$ and $\sigma = 1$, calculate the probability that a servo will last for 40 hours.

Parameter Estimation

Q3.1:

Use maximum likelihood estimation to fit a log-normal PDF to the failure data provided in Table 2. Calculate the 95% confidence bounds in the MLE estimates and the probability that the component will fail between 30 and 55 years.

Table 2: Failure times, t , in units of years.

49.5742	39.0231	28.241	15.4782	16.6129	56.0018	39.1695	23.3995	9.34463
25.07	16.3242	7.1327	24.5379	95.74	13.3444	45.9901	22.8084	45.0592
49.9091	10.9584	22.0764	20.182	19.9721	14.9876	12.6724	17.3804	5.13221
21.0974	13.3389	12.4727	14.2646	17.2633	14.3847	17.5576	25.6905	7.18939
33.0662	19.2535	13.5511	20.9923	16.0762	24.9305	12.2074	5.23867	18.3745
16.6716	40.6632	18.8055	12.6727	11.24	21.6383	14.7064	17.601	15.0596
32.7493	18.8793	15.8212	6.69836	11.7989	27.7205	18.9279	72.2692	8.4983
18.6928	13.4246	38.2967	32.851	13.5905	30.0529	15.1524	60.4526	18.3166
5.44084	6.10256	20.3197	8.4967	35.7099	11.3333	31.722	18.1207	11.6619
15.0325	25.5701	20.1378	26.1686	12.1678	38.3752	13.6005	6.39596	10.8213

Q3.2:

15 components are tested, 12 of which have failure times recorded while 3 did not fail during the 24 hours in which the experiment was run. Failures are assumed to follow an exponential distribution and those failure times recorded are presented in Table 3 below. Using all of the data (censored and uncensored) calculate $\hat{\lambda}$, assuming the data follows an exponential distribution.

Q3.3:

A technician performs a series of 25 failure tests on a component. The tests are run for a total of 7 hours. At the end of the test 1 component has not yet failed. The technician also did not record an exact failure time for 4

Table 3: Failure times, t , in units of hours.

5.9653	2.3421	0.7956	0.1604	0.0151	3.0238
4.2605	1.2375	8.6255	6.8242	0.7645	5.8619

components which failed during the first 0.25 hours of the test. Given the 20 recorded failure times (in hours) presented in Table 4 and the knowledge that 4 components failed before 0.25 hours and 1 component hadn't failed after 7 hours calculate the maximum likelihood estimate of $\hat{\lambda}$ assuming that the distribution is exponential in nature.

Table 4: Failure times, t , in units of hours.

0.2967	0.9752	1.6013	3.1594	4.6646
0.3594	1.3873	1.6849	3.4113	5.0373
0.406	1.4796	2.2832	3.6384	5.0554
0.5223	1.5322	2.7337	3.7169	6.4226

Q3.4:

A component is tested 100 times and found to fail via two different modes. Only the failure times were recorded and not the method of failure. Use the provided in Table 5 failure data to determine maximum likelihood estimates of the parameters for the two failure modes. Assume that both modes are described by a normal distribution.

Table 5: Failure times, t , in units of hours.

18.535	22.285	24.571	10.066	13.913	18.24	13.212	14.701	3.244	24.081
20.815	20.745	23.06	13.545	14.013	20.095	19.904	20.618	17.545	19.91
24.038	22.995	13.089	17.644	18.303	15.742	17.465	16.507	19.377	6.892
20.169	12.192	4.014	22.486	12.649	14.417	18.51	15.565	16.24	16.048
15.477	24.023	24.262	21.442	12.734	22.303	16.669	19.36	17.681	14.762
23.683	21.824	21.54	10.255	3.535	16.264	5.495	22.827	9.165	19.672
22.194	6.539	19.132	23.186	13.552	20.489	18.067	2.62	11.679	17.566
20.5	22.055	16.361	19.213	20.673	12.763	23.324	22.802	13.702	17.186
23.366	22.875	9.023	22.969	17.805	18.051	22.667	12.448	21.383	14.231
15.697	22.388	20.439	11.142	17.911	22.624	16.549	22.821	15.19	18.581

Multi-variate Distributions

Q4.1:

Consider the observations in Table 6 below. Fit a two dimensional normal distribution to the data set. What are the MLE parameters?

Table 6:

x_1	x_2
0.8265	1.7644
0.8576	1.661
1.7552	2.3293
1.0233	2.3513
1.3421	1.5131
0.8821	2.2614
0.7454	1.7379
1.0525	1.1175
1.238	1.3376
1.0518	2.3298
1.5904	1.8017
1.071	1.8711
0.9222	1.183
0.6637	2.019
1.1401	1.8232

Q4.2:

Consider the manufacture of the flange highlighted in Figure 1 below. Upon manufacture measurements are taken of the width and height of a series of flanges and a two dimensional normal distribution is fitted to the data with the following parameters, where parameter one defines the height of the flange and parameter two defines the width of the flange.

$$\boldsymbol{\mu} = \begin{bmatrix} 8.0081 \\ 1.9591 \end{bmatrix}$$

$$\boldsymbol{\Sigma} = \begin{bmatrix} 0.0737 & 0.0136 \\ 0.0136 & 0.0714 \end{bmatrix}$$

If the flange width falls below 1.4mm or the flange height falls below 7.5mm then the casing has to be scrapped. If the flange width is above 2.4mm or

the flange height is above 8.5mm then the casing has to be machined again. Calculate the probability of a casing being scrapped? Calculate the probability of a casing being reworked?

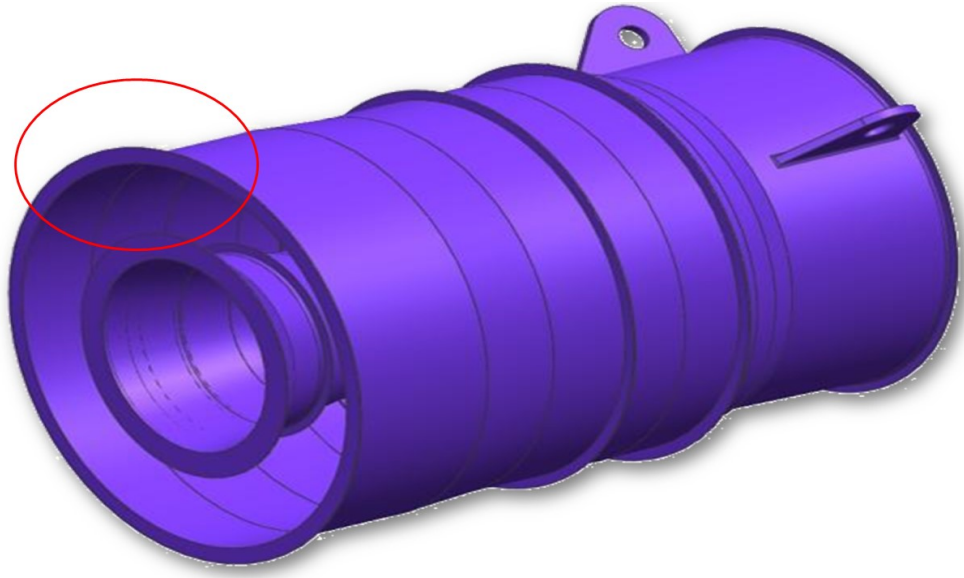


Figure 1: Gas turbine compressor intercase with end flange highlighted.

Reliability Modelling

Q5.1:

Simplify the reliability block diagrams presented in Figure 2.

Q5.2:

Calculate the reliability of the system illustrated in Figure 3.

Q5.3:

Consider a microcar with the reliability block diagram defined in Figure 4. The reliability of each individual component is assumed to be approximated by the Weibull reliability function,

$$R(t) = e^{-\left(\frac{t-\gamma}{\eta}\right)^\beta},$$

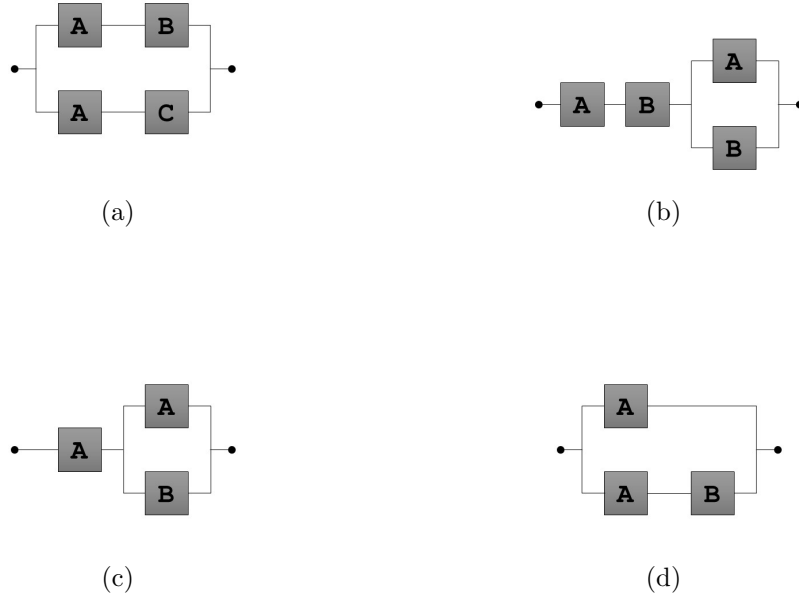


Figure 2: Reliability block diagrams for simplification.

when $t \geq \gamma$ and 1.0 otherwise. As usual β defines the shape parameter, η the scale parameter and γ is the failure free time. The parameters for each component are given below in Table 7. Calculate the reliability of the car and each sub-system in steps of 500 hours from $t = 1000$ to $t = 10,000$. Which sub-system drives the overall reliability? How would you improve the reliability of the car?

Q5.4:

A single component sub-system can have redundancy introduced in one of two ways, either active or inactive redundancy (i.e. hot or cold redundancy). Calculate and compare the reliability of the system in these two configurations and when there is no redundancy at $t = 750$. The component is assumed to have failures which are a function of t and are described by a normal distribution with $\mu = 550$ and $\sigma = 200$.

Q5.5:



Table 7: Microcar Weibul parameters.

Component	β	η	γ
Engine	3.25	5500	8500
Steering	3.7	3000	7500
Brakes	4	2250	7500
Transmission	3.5	2500	9000
Tyre	2.6	1250	4250
Seat	1.9	2000	5500
Boot	1.4	4100	6000
Roof Rack	1.7	2300	4500
Tow Bar	3.1	2250	4200

The tracks, wheel and conveyor belts of a bucket excavator are powered by a series of electric motors which require a total of 6.0MW of power to keep running. The electricity for these motors is provided by 4 identical diesel generators on board the excavator whose failures are a function of the power they produce and described by a three parameter Weibull distribution with $\beta = 3.0$, $\eta = 2.2$ and $\gamma = 0.5$. Calculate the reliability of this system.

Q5.6:

The reliability of an unmanned system is assumed to be driven by four sub-systems in series, the battery, speed controller, electric motor and propeller pitch servo. For each sub-system the designer has a choice of components which meet the performance requirements however, each component has a different reliability and cost (see Table 8 below). What is the cost of the most reliable system? What is the reliability of the cheapest system? What is the most reliable system achievable for a maximum of £60?

Robust Design

Q6.1:

A function $f(\mathbf{x})$ is defined by two variables x_1 and x_2 via the following equation,

$$f(\mathbf{x}) = \left(x_2 - \frac{5.1}{4\pi^2} x_1^2 + \frac{5x_1}{\pi} - 6 \right)^2 + 10 \left(1 - \frac{1}{8\pi} \right) \cos(x_1) + 5x_1 + 10,$$

Table 8: Unmanned system component reliabilities and costs.

Sub-system	Option No.	Reliability	Cost (£)
Battery	1	0.89	5.50
	2	0.91	8.50
	3	0.975	11.00
	4	0.8	3.50
Speed controller	1	0.95	9.00
	2	0.965	12.00
	3	0.999	20.00
	4	0.975	15.50
Electric motor	1	0.995	30.00
	2	0.965	17.50
	3	0.985	22.50
	4	0.99	27.00
Servo	1	0.801	4.00
	2	0.998	17.50
	3	0.988	6.50
	4	0.757	5.25

where $x_1 \in [-5, 10]$ and $x_2 \in [0, 15]$. Calculate the minimum of the above function within the given bounds of x_1 and x_2 and its location.

Q6.2:

In the previous question it was assumed that the values of x_1 and x_2 are deterministic. Given that these variables are subject to variations defined by normal distributions with $\sigma_1 = 3.0$ and $\sigma_2 = 1.5$ respectively and with mean corresponding to x_1 and x_2 . Determine the values of x_1 and x_2 which correspond to a robust optimum where the mean of $f(\mathbf{x})$ subject to the above variations is to be minimised.

Optimal Maintenance Scheduling

Q7.1:

A solenoid valve in an aircraft fails according to a log-normal distribution with $\mu = 6$ and $\sigma = 2$. The cost of preventative replacement is \$1000 and

failure replacement is \$10,000. Calculate the constant interval replacement cost, $c(t_p)$, and replacement at predetermined age cost, $c(t_{pa})$, at 2000 hours.

Q7.2:

Determine the optimal constant interval replacement policy for a single turbine stage where one stage comprises of 30 turbine blades, one disc and 30 stator vanes. The failure probabilities for each component are described by PDF parameters in Table 9. Assume that the cost of preventative replacement is \$11,000 and the cost of replacement upon failure is \$200,000.

Table 9: Weibull parameters describing the failure probability of individual components in a gas turbine stage.

Component	β	η
Turbine Blade	2.55	115000
Turbine Disc	4.40	2.1×10^6
Turbine Stator Vane	3.1	500,000

Project Uncertainty Management

Q8.1:

The manager of a small charter aircraft company is considering three bids from separate maintenance providers for the contract to service his fleet of aircraft. The terms of each of the three contracts are outlined below in Table 10. Each contract comprises of a fixed initial fee plus a fee per mile flown by the aircraft. The final cost of maintaining a single aircraft will therefore be the initial fee plus the fee due to the miles flown since the previous service. The manager elicits information on the likely number of miles flown by an aircraft resulting in a minimum (0%) mileage of 5,000 miles and a maximum (100%) of 15,000 miles. Which contract should be accepted and why? Assume a uniform distribution function.

Q8.2:

The final assembly manager for a small satellite system has been asked to assess the potential impact of a delay in final delivery. The contract to deliver the satellite system to the customer for launch stipulates that there is

Table 10: Proposed contract terms.

	Fixed fee (\$)	\$/mile
Company 1	7,500	5.00
Company 2	12,000	4.25
Company 3	30,000	2.00

a considerable penalty of \$50k for every day the system is late. Eliciting his team, the manager has determined that a minimum of delay of -2 days (i.e. 2 days ahead of schedule) and a maximum delay of 8 days. What is the probability of a delay occurring? What is the expected delay and the expected cost to the satellite manufacturer? Assuming a uniform distribution function is defined as part of the elicitation.

Q8.3:

The assembly process for a business jet comprises of four sequential tasks, listed in Table 11. In order to meet the contracted delivery time the aircraft's assembly should be completed within 80 days. Any delay to delivery comes at a penalty of \$5k per day. Given the uncertainties for the completion times defined in Table 11 (units of days) what is the expected delay cost? The assembly manager has the option of crashing task D at a cost of \$5k, the new completion uncertainty is also indicated in Table 11. Comment, is crashing worth it?

Table 11: Aircraft assembly tasks and their uncertainty.

Task	Description	Distribution
A	Fuselage assembly	Uniform in the range 30-40
B	Wing assembly	Triangular, min at 18, peak at 20, max at 28
C	Engine assembly	Gaussian, $\mu = 15$, $\sigma = 2$
D	Avionics & interior	Triangular, min at 5, peak at 10, max at 12
$D_{crashed}$	Avionics & interior	Triangular, min at 2, peak at 5, max at 6

Appendix

