(EF/TN/A/47) TWO CALCULATION PROCEDURES FOR STEADY, THREE-DIMENSIONAL FLOWS WITH RECIRCULATION (Imperial Coll. of Science and Technology) 25 p HC $3.25

N73-22238

Unclas
H2/12 69688

IMPERIAL COLLEGE OF SCIENCE AND TECHNOLOGY

Department of Mechanical Engineering

Exhibition Road, London, S.W.7.

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ABSTRACT

Two procedures are described for solving the Navier-Stokes equations for steady, fully three-dimensional flows: both are extensions of earlier methods devised for three-dimensional boundary layers, and have the following common features: (i) the main dependent variables are the velocities and pressure; (ii) the latter are computed on a number of staggered, interlacing grids, each of which is associated with a particular variable; (iii) a hybrid central-upwind difference scheme is employed; and (iv) the solution algorithms are sufficiently implicit to obviate the need to approach the steady state via the time evolution of the flow, as is required by wholly explicit methods.

The procedures differ in their manner of solving the difference equations. The SIVA (for Simultaneous Variable Adjustment) procedure, which is fully-implicit, uses a combination of algebraic elimination and point-successive substitution, wherein simultaneous adjustments are made to a

point pressure, and the six surrounding velocities, such that the equations for mass and (linearised) momentum are locally satisfied.

The SIMPLE (for Semi-Implicit Method for Pressure-Linked Equations) method proceeds in a successive guess-and-correct fashion. Each cycle of iteration entails firstly the calculation of an intermediate velocity field which satisfies the linearised momentum equations for a guessed pressure distribution: then the mass conservation principle is invoked to adjust the velocities and pressures, such that all of the equations are in balance.

By way of an illustration of the capabilities of the methods, results are given of the calculation of the flow of wind around a building, and the simultaneous dispersal of the effluent from a chimney located upstream.

1. Introduction

1.1 Objectives of the present research.

We are here concerned with prediction methods for that class of convective-flow phenomena which are steady, recirculating, low-speed and three-dimensional: the majority of the practically-important flow situations encountered in industrial, environmental, physiological and other fields are of this kind. Two calculation procedures for such flows will be described: both proceed by way of finite-difference solution of the Eulerian partial-differential equations for the conservation of mass, momentum, energy and other properties; and both employ the velocities and pressure as the main hydrodynamic variables.

1.2 Relation to previous work.

Although there exist a number of finite-difference procedures which could, in principle, be used for the present class of problems, none appear to be well-suited for this purpose. Thus, for example, nearly all of the available methods attempt to follow the time evolution of the flow in arriving at the steady-state solution. When however the latter is the *only* feature of interest, this is usually needlessly expensive, especially when an *explicit* formulation is employed.

The procedures to be described here contain a number of innovations, allowing particularly economical routes to the steady state: they also however incorporate many known features including:- the displaced grids for velocity and pressure employed by Harlow and Welch (1965); the concept of a guess-and-correct procedure for the velocity field, used by Amsden and Harlow (1970) and Chorin (1968); and the implicit calculation of velocities, along the lines of the Pracht (1970) version of the Harlow-Welch (1965) procedure. Additional guidance in the formulation of the new procedures has has been derived from earlier work by the authors and their colleagues on methods for two-dimensional flows (Patankar and Spalding, 1970; Gosman et al, 1969), and three-dimensional boundary layers (Patankar and Spalding, 1972a;Caretto et al, 1972).

1.3 Contents of the paper.

Section 2 of the paper is devoted to the description of the two procedures, code-named SIMPLE and SIVA. Because the point of departure between the two methods is in the manner of solving the finite-difference equations, the latter are described first; then details are given of the individual solution paths.

In Section 3, we provide a summary of the experience gained from application of the procedures to a variety of test cases. Then, by way of a demonstration, we present the results of a computer simulation of the flow of wind past a building, and the simultaneous dispersal of the effluent from a chimney located upwind of the building. Finally, in Section 4 we present our conclusions about the relative merits of the two procedures, and the prospects for further development.

2. Analysis

2.1 The equations to be solved.

The mathematical problem may be compactly expressed, with the aid of Cartesian tensor notation, in terms of the following set of differential equations:

$$\frac{\partial}{\partial x_i}(\rho u_i) = 0 \quad ; \qquad (1)$$

$$\frac{\partial}{\partial x_i}(\rho u_i u_j) - \frac{\partial}{\partial x_i}\left(\mu_{eff}\frac{\partial u_j}{\partial x_i}\right) - \frac{\partial P}{\partial x_j} + s_j = 0 \quad ; \qquad (2)$$

$$\frac{\partial}{\partial x_i}(\rho u_i \phi) - \frac{\partial}{\partial x_i}\left(\Gamma_{\phi,eff}\frac{\partial \phi}{\partial x_i}\right) + s_\phi = 0 \quad ; \qquad (3)$$

which express the laws of conservation of mass, momentum and a scalar property ϕ respectively. Here the dependent variables are the (time-average) values of: the velocities u_j; the pressure P; and ϕ, which stands for such scalar quantities as enthalpy, concentration, kinetic energy or dissipation rate of turbulence (Launder and Spalding, 1971) and radiation flux (Spalding, 1972) etc. The symbols s_j and s_ϕ stand for additional sources (or sinks) associated with such phenomena as natural convection, chemical reaction and non-uniformity of transport coefficients, while ρ, μ_{eff} and $\Gamma_{\phi,eff}$ are respectively the density, viscosity and exchange coefficient for ϕ. The subscript 'eff' appended to the latter two indicates that, for turbulent flows, they are sometimes ascribed 'effective' values, deduced from turbulence quantities.

In order fully to appreciate the extent of the mathematical problem, it is necessary to observe that, for many practically-interesting situations the number of equations may be very large indeed. Thus, for a power-station-furnace calculation, the 4 equations represented by (1) and (2) must be supplemented by at least 7 others, comprising 2 for turbulence properties, 1 each for energy and concentration, and 3 for radiation fluxes. The need for solution procedures which are economical of both computer storage and computing time is therefore obvious.

2.2 Finite-difference equations.

(a) Grid and notation. The staggered-grid system employed for both methods is depicted in Fig. 1: this shows only the xy plane, but the treatment in the other planes follows identical lines.

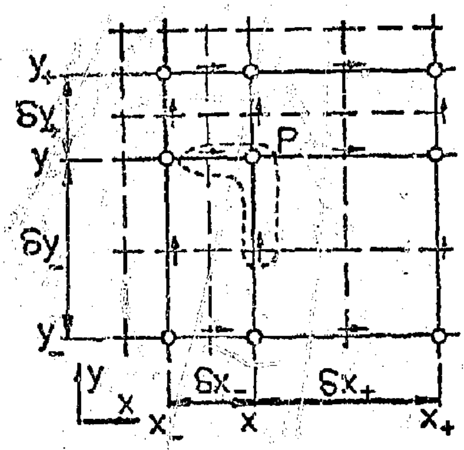

Fig. 1. Illustration of staggered-grid system.

The intersections of the solid lines mark the grid nodes, where all variables except the velocity components are stored. The latter are stored at points which are denoted by the arrows and located mid-way between the grid intersections. This arrangement has two especial merits: firstly, it places the velocities between the pressures which drive them; and secondly, these velocities are directly available for the calculation of the convective fluxes across the boundaries of the control volumes (dashed lines) surrounding the grid nodes.

A considered node and its immediate neighbours are denoted by the subscripts P, x+, x-, y+, y-, z+ and z-: the significance of these can be perceived from Fig. 1. The velocities are similarly referenced, with the convention that P (and each of the other subscripts) now refers to a cluster of variables, as indicated in the diagram.

(b) Differencing practices. Attention will first be focussed on the differential conservation equation (3) for a scalar property ϕ. A difference equation relating ϕ_P to the surrounding ϕ's is obtained by integration of (3) over the control volume enclosing P, with the aid of flux expressions derived from one-dimensional flow theory. Some details will now be given.

We represent the nett x-direction convection and diffusion of ϕ through the control volume (Fig. 2) by:

$$C^{\phi}_{x+}(\phi_{x+}-\phi_P) + C^{\phi}_{x-}(\phi_{x-}-\phi_P) \qquad (4)$$

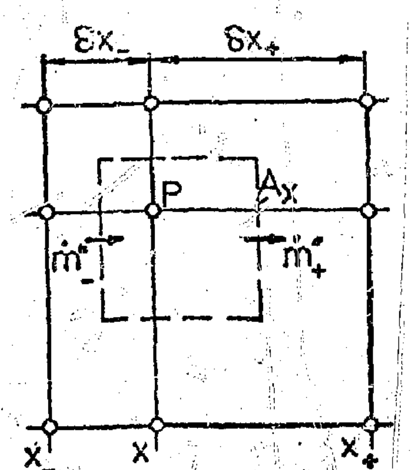

Fig. 2. Notation for the x-direction flux expressions.

where, e.g.:

$$C^{\phi}_{x+} = \begin{cases} 0, \text{ when } F_{x+} > D_{x+}; \\ -2F_{x+}, \text{ when } F_{x+} < -D_{x+}; \\ D_{x+} - F_{x+}, \text{ in all other circumstances.} \end{cases}$$

$$F_{x+} \equiv \dot{m}''_{+} A_x/2 \quad ;$$

$$D_{x+} \equiv \overline{\Gamma}_{\phi+} A_x/\delta x_{+};$$

and $\dot{m}''_{+}$, A_x and $\Gamma_{\phi+}$ respectively stand for the mass flux, cross-sectional area and average exchange coefficient at the boundary in question. The other quantities in (4) are similarly defined.

The above expression may be regarded as a hybrid of central- and upwind-difference schemes, in that it reduces to the former when the ratio $|F/D|$ (a local Peclet number) is less than unity; and it yields the large- (F/D) asymptote of the latter for $|F/D|$ greater than unity. The hybrid scheme has the advantages of being more accurate over a wide range of F/D (Spalding, 1970; Runchal, 1970), than either of its components, and of yielding a diagonally-dominant matrix of coefficients for all F/D.

(c) The difference equations. When the fluxes in the y and z directions are expressed in a similar manner, the resultant finite-difference equation is:

$$C_P^{\phi}\,\phi_P = C_{x+}^{\phi}\,\phi_{x+} + C_{x-}^{\phi}\,\phi_{x-} + C_{y+}^{\phi}\,\phi_{y+} + C_{y-}^{\phi}\,\phi_{y-} + C_{z+}^{\phi}\,\phi_{z+} + C_{z-}^{\phi}\,\phi_{z-} + S^{\phi} \quad ; \qquad (5)$$

where S^{ϕ} represents the integral of the source s_{ϕ} over the control volume and:

$$C_P^{\phi} \equiv C_{x+}^{\phi} + C_{x-}^{\phi} + C_{y+}^{\phi} + C_{y-}^{\phi} + C_{z+}^{\phi} + C_{z-}^{\phi} \quad .$$

The treatment of the momentum equations (2) differs in no essential way from that above. The control volumes for the velocities are of course displaced from those for ϕ, but this presents no new problems, apart from the minor one of the necessity sometimes to interpolate in order to obtain convection velocities, densities, viscosities etc. at the required locations. These are matters of detail to which space cannot be given here: however it should be stated that our choice of interpolation practices is guided by the requirement that the resulting difference equation is conservative.

If we denote the velocities in the x,y and z co-ordinate directions by u, v and w respectively, then the difference equations for momentum may be written:

$$C_P^u\,u_P = \sum_n C_n^u\,u_n + A_x\,(P_{x-} - P_P) + S^u \quad ; \qquad (6)$$

$$C_P^v\,v_P = \sum_n C_n^v\,v_n + A_y\,(P_{y-} - P_P) + S^v \quad ; \qquad (7)$$

$$C_P^w\,w_P = \sum_n C_n^w\,w_n + A_z\,(P_{z-} - P_P) + S^w \quad . \qquad (8)$$

Here, the summations are over the six neighbouring velocities; and the coefficients in the equations are defined in an analogous fashion to those in (5).

Finally, we complete the transformation to difference form by expressing the continuity relation (1) as:

$$[(\rho u)_{x+} - (\rho u)_P] A_x + [(\rho v)_{y+} - (\rho v)_P] A_y$$
$$+ [(\rho w)_{z+} - (\rho w)_P] A_z = 0 . \qquad (9)$$

2.3 The solution procedures.

(a) Appreciation of the task. The most obvious, and perhaps most important, obstacle to be overcome in the solution of the hydrodynamic equations is the presence of the unknown pressures on the right-hand sides of equations (6) to (8): for if these happened to be known, the equations for u, v and w would have the same form as that for ϕ; and many satisfactory techniques are available for the solution of the latter. There are, it is true, other obstacles, such as the non-linearities and interlinkages introduced by way of the coefficients and source terms; but these can be usually overcome without too much difficulty. We shall therefore focus attention, in the paragraphs to follow, on ways of overcoming the problem of the velocity-pressure interlinkage.

(b) The SIMPLE procedure. This 'Semi-Implicit Method for Pressure-Linked Equations' solves the set (6) to (9) by a cyclic series of guess-and-correct operations, wherein the velocities are first calculated by way of the momentum equations for a guessed pressure field, and then the latter, and later the velocities, are adjusted so as to satisfy continuity.

The first step in the cycle is straightforward: thus the guessed pressures (which may be initial guesses, or values from a previous cycle), denoted by P^*, are substituted into linearised* versions of (6) - (8). These are then solved to yield a field of intermediate velocities u^*, v^* and w^* which will not, unless the solution has been reached, satisfy continuity.

It is here that the main novelties of the procedure enter, in the manner of satisfying the continuity requirement. The approach is to substitute for the velocities in eqn. (9) relations of the form:

$$u_P = u_P^* + A_P^u(P'_{x-} - P'_P) \quad ; \qquad (10)$$

$$v_P = v_P^* + A_P^v(P'_{y-} - P'_P) \quad ; \qquad (11)$$

$$w_P = w_P^* + A_P^w(P'_{z-} - P'_P) \quad ; \qquad (12)$$

where P' is a pressure correction, and the A's bear the following relation to coefficients in the momentum equations:

$A_P^u \equiv A_x/C_P^u$; $A_P^v \equiv A_y/C_P^v$; and $A_P^w \equiv A_z/C_P^w$.

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*The coefficients and source terms are evaluated from the previous cycle, and held constant.

The result is the finite-difference equivalent of a Poisson equation for P', viz:

$$C_P^P \; P'_P \; = \sum_n C_n^P \, P'_n + S^P \quad . \tag{13}$$

Here the summation sign has the usual meaning, and the coefficients are given by:

$$S^P \equiv [(\rho u^*)_{x+} - (\rho u^*)_P] \, A_x + [(\rho v^*)_{y+} - (\rho v^*)_P] \, A_y$$

$$+ [(\rho w^*)_{z+} - (\rho w^*)_P] \, A_z \quad ;$$

$$C_P^P \equiv \sum_n C_n^P \quad ;$$

$$C_{x-}^P \equiv \rho_{x-} \, A_x A_P{}^u \quad ;$$

with similar definitions for the other terms. S^P, it should be noted, is nothing more than the local mass imbalance of the intermediate velocity field: so, when continuity is everywhere satisfied, the pressure correction goes to zero, as would be expected.

Once the P' field has been obtained from (13), it is a straightforward matter to update the pressures and velocities (from eqns. 10-12): then, if necessary, they may be used as guesses for a new cycle. If there are ϕ's to be calculated, they may be fitted in at a convenient stage in the cycle: often the choice is arbitrary.

In view of the central role which the P' equation plays in the calculation, some further details about its derivation and application will now be given. It is based in part on equations (10) - (12), which are in turn derived from the momentum equations on the assumption that the velocities are influenced more by pressure changes than by any other cause. The point to note is that this assumption is without influence on the final solution; for as this is approached, P' goes to zero.

A particular merit of working with P' is the ease with which pressure boundary conditions may be specified: thus, for example, at prescribed-pressure boundaries, P' is simply set to zero; while at impermeable walls the appropriate condition is one of zero normal gradient, as reference to eqns. (10)-(12) will confirm.

The final point to be made about the SIMPLE procedure is that, because it computes the variable fields successively, rather than simultaneously, it is highly flexible in respect of the methods of solution which it will admit for the difference equations. For the present calculations, we have employed a line-iteration method, wherein the unknown variables along each grid line are calculated by application of the tri-diagonal matrix algorithm, on the assumption that values on neighbouring lines are known. This operation is performed in turn on the sets of lines lying in the x, y and z directions: it usually suffices to perform one such 'triple sweep' on the velocities and ϕ's, and three sweeps on P', per cycle of calculation. This method is substantially faster than point iteration; however it must be stressed that when even more economical methods become available, they may readily be incorporated into the procedure.

(c) The SIVA procedure. This procedure derives its name from the novel way in which it combines point iteration with SImultaneous Variable Adjustment. With this combination, it is possible to satisfy simultaneously, on a local basis, the equations for momentum and continuity: although this balance is later destroyed when neighbouring nodes are visited, the nett effect is to reduce the residual sources, and so procure convergence.

The procedure involves the adjustment, as each node is visited, of 7 variables (excluding the ϕ's), namely the pressure P, and the 6 surrounding velocities, u_P, u_{z+}, v_P, v_{y+}, w_P and w_{z+}. The formulae for the variable adjustments are obtained by algebraic solution of: the continuity equation (9); and linearised versions of the momentum equations for the six velocities, expressed in the following form:

$$u_P = \alpha_P^u u_{x+} + \beta_P^u p_P + \gamma_P^u \quad ; \qquad (15)$$

$$v_P = \alpha_P^v v_{y+} + \beta_P^v p_P + \gamma_P^v \quad ; \qquad (16)$$

$$w_P = \alpha_P^w w_{z+} + \beta_P^w p_P + \gamma_P^w \quad ; \qquad (17)$$

with similar expressions for u_{x+}, v_{y+} and w_{z+}. The quantities α, β and γ in these equations are readily deducible from the parent equations (6)-(9), whose terms involving variables outside of the 'SIVA cluster' have been swept into the γ's, and regarded (temporarily) as knowns. It is a straightforward matter to manipulate this set into equations which contain only the known coefficients on the right-hand sides: details will not be given here.

SIVA proceeds in all other respects in the manner of a normal point-iteration procedure: thus the grid is repeatedly swept, until the residual sources of the difference equations are reduced to acceptably small values. As with the SIMPLE method, the calculation of ϕ's is fitted in where appropriate.

3. Applications.

3.1 Test calculations.

The SIMPLE and SIVA procedures were initially tested by application to a class of problems involving the laminar motion of a fluid in an enclosure of side H, which has one wall moving at a steady velocity V in its own plane.

Calculations were first made on the two-dimensional version of this problem, for comparison with earlier work. The present methods were found to compare favourably: thus, for a coarse mesh of 10 equally-spaced intervals, convergent solutions were obtained for Reynolds numbers (based on V and H) in excess of 10^6; and the results for a Reynolds number of 100 agreed to within a few percent with Burggraf's (1966) fine-mesh calculations. The SIMPLE procedure did exhibit some signs of instability in the initial stages of the calculations at the higher Reynolds numbers: this however could easily be cured by straightforward under-relaxation of P' (with a factor of about 0.2), often in the initial stages only.

The results obtained for the three-dimensional case were equally encouraging. Thus, no deterioration in numerical stability was observed to result from the introduction of the third dimension; and, although no other solutions were available for comparison, the predictions were entirely plausible.

Although the initial studies confirmed that the two methods gave equal accuracy and numerical stability, the SIMPLE method proved to be appreciably more economical of computing time than SIVA. It is therefore the former which we currently favour in our work.

In subsequent studies, SIMPLE has been successfully applied to several problems of practical interest, including the prediction of flow, heat transfer and chemical reaction in a three-dimensional furnace (Patankar and Spalding, 1972b) and the calculation of the steady-state and transient behaviour of a shell-and-tube heat exchanger (Patankar and Spalding, 1972c). Flows with strong effect of compressibility, and with distributed internal resistances, have also been predicted by the SIMPLE method.

3.2 The building problem.

As a further example of the type of problem for which the SIMPLE method is well-suited, we here present calculations of the simulated (laminar) flow of wind past the slab-sided 'building', depicted in Fig. 3. The oncoming wind varies in strength in a parabolic fashion with distance from the ground, and is directed normal to the face of the building. An additional feature is a chimney located upwind of the building: the path of the effluent from this is also followed numerically.

The grid employed for the calculations had 10 nodes in each direction: non-uniform spacing was employed so as to cause the nodes to be concentrated near the building, and more widely-spaced elsewhere. The domain of solution, measured in building heights H, extended approximately $\pm$ 5H in the mainstream (z) direction, and 8H in both the vertical (y) and lateral (x) directions. The plane x=0 was prescribed as a plane of symmetry, while at all other free boundaries

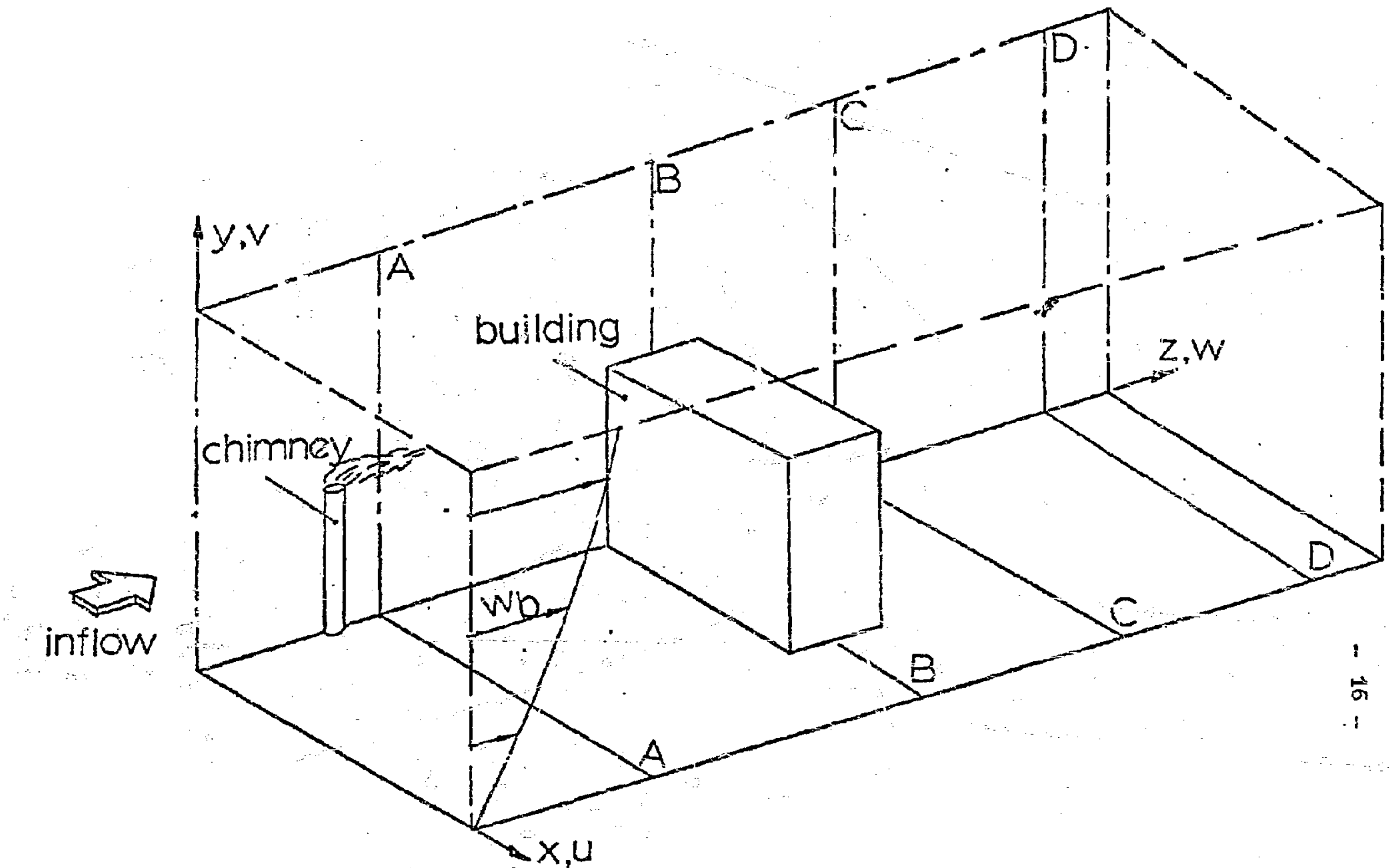


Fig. 3. Illustration of the flow-past-building problem.

the flow was presumed to be undisturbed by the presence of the building. The Reynolds number, based on H and the undisturbed velocity w_B at y=H, was approximately 100, in this purely illustrative example.

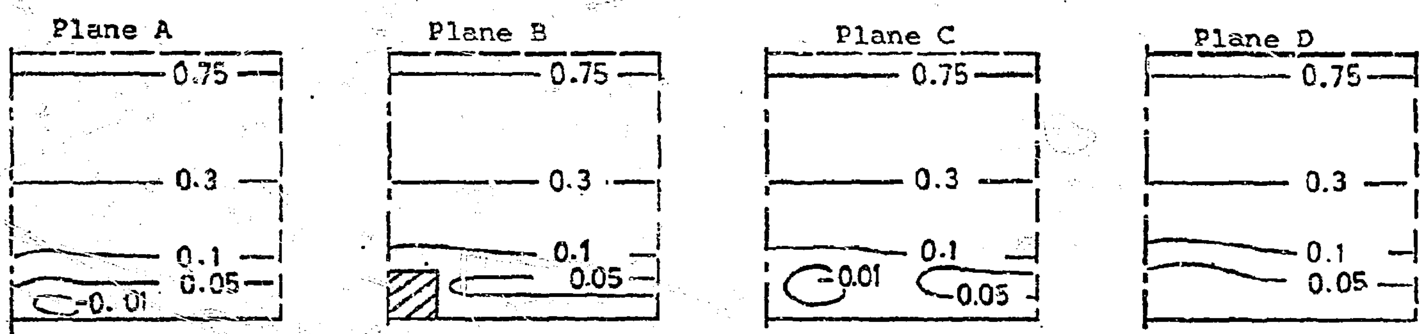
The results are displayed in Fig. 4, in the form of plots, at a number of constant-z planes, of: contours of constant mainstream velocity; vectors representing the direction and magnitude of the resultant velocities in the xy planes; isobars; and contours of the effluent concentration.

Taken together, the velocity and pressure plots reveal a consistent and plausible pattern of behaviour: thus the build-up of pressure in front of the building provokes reverse flow (indicated by the negative-w contour) in the low-velocity region near the ground, and deflects the wind away from the building. Downstream, the low-pressure zone behind the building also gives rise to reverse and lateral flows: now however the fluid is drawn inwards.

The concentration contours show that the effluent plume initially spreads downwards, thereby causing relatively high concentrations at the upwind face of the building. The flow around the latter then deflects the plume upwards, so that the concentration on the downwind face is lower, although still appreciable.

Although it cannot be claimed that a laminar-flow calculation on a relatively sparse grid is quantitatively representative of the real situation, the above results are probably at least qualitatively correct: moreover, they were obtained at a quite modest cost (approximately 100 seconds on a CDC 6600 machine).

(a) Main-flow velocity contours (w/w_{max}).



(b) Resultant velocity in cross-stream planes.

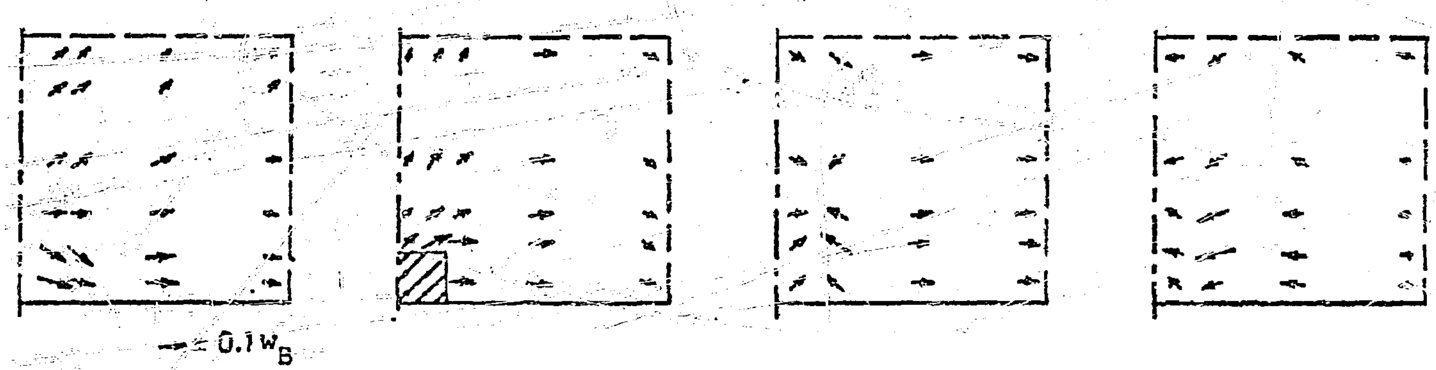
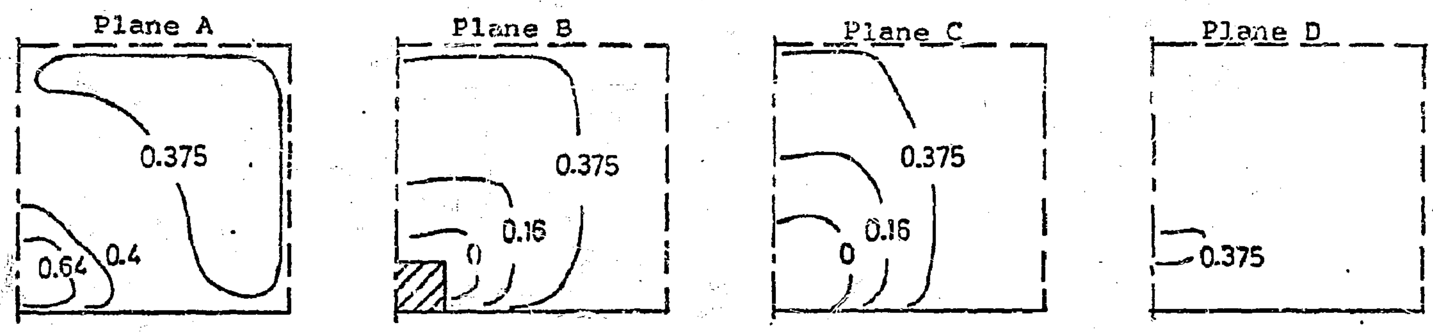


Fig. 4 Predictions for the building problem.

(c) Static pressure contours $[2(P-P_{ref})/\rho w_B^2]$



(d) Effluent concentration (concentration at source = 100)

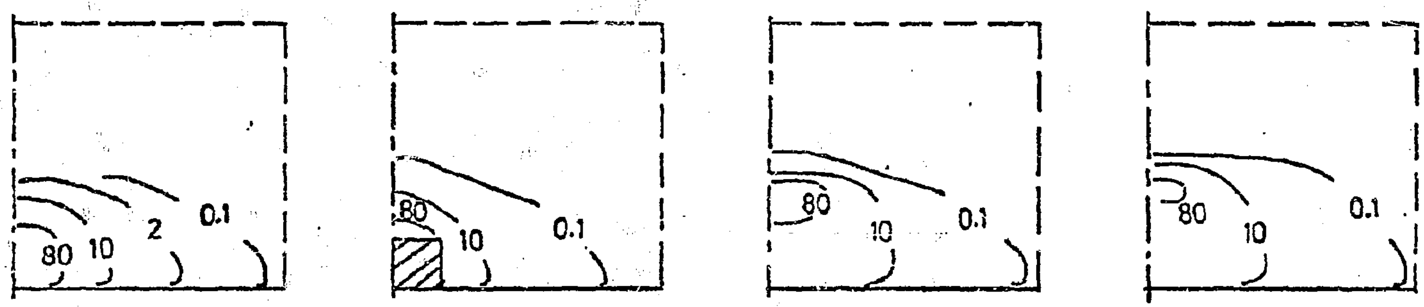


Fig. 4 (Cont'd).

4. Discussion and conclusions.

4.1 Assessment of the procedures.

Experience with the SIVA and SIMPLE algorithms, which have now been applied to a large number of flow situations of varied type, has demonstrated the great flexibility and stability that results from using implicit finite-difference formulations, with the hybrid difference scheme. It has also shown that the line-by-line nature of the SIMPLE adjustment procedure makes for greater economy of computer time than the point-by-point SIVA adjustment. The slightly-reduced stability of SIMPLE can be rectified by an inexpensive under-relaxation. The authors therefore intend to concentrate on SIMPLE in their future work.

4.2 Prospects for future development.

The example of Fig. 3 shows that the calculation procedure can be employed for predicting practically-important phenomena which, at present, can be predicted only by way of rather expensive and time-consuming experiments. However, a consideration of the shortcomings of that example shows also much development still to be done. First of all, the calculation was performed for a low-Reynolds-number laminar flow; but flows over real buildings are of high Reynolds number, and turbulent. It is therefore necessary to incorporate into the calculation procedure "turbulence models", of the kind recently surveyed by Launder and Spalding (1971).

Secondly, it will have been observed that the calculation task was made especially easy by the fact that four of the boundaries of the domain of integration were treated as impervious to matter, while the inlet boundary was as one at which the velocity distribution was known. In reality, the elliptic nature of the flow ensures that the presence of the building modifies the velocity distribution at these boundaries: some economical means of calculating this modification needs to be built into the calculation procedure.

Finally, buildings are not simply rectangular blocks; sometimes the departures from simplicity of form may have significant aerodynamic effects. It is therefore necessary to arrange that significant minor details of the surface, for example its distribution of roughness, can be allowed with the calculation scheme, without necessitating excessive refinement of the grid.

If these problems can be speedily surmounted, there is every reason to expect that numerical computations will replace model experiments for civil-engineering aerodynamics, furnace design, and many areas of hydraulic and aeronautical engineering. No difficulties of principle appear to stand in the way of these developments, and none of the difficulties of detail is of a kind which has not been surmounted elsewhere.

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